SELF-OSCILLATORY SYSTEM SUBJECTED TO AN EXTERNAL THERMAL AGENT

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The conditions for the onset of oscillations in a system are analyzed for the case in which the only external agent is a thermal agent. The stability boundary and the dependence of the oscillation frequency on the parameters of the system are evaluated. An experimental apparatus is described, and the results found in experiments with moist water vapor are described.

The oscillations of parameters which occur in the operation of heat-transfer systems involving steam generation, described in [1-3], are of definite interest both in connection with the goal of achieving maximum utilization of steam-generating systems and in connection with the development of new systems for converting thermal energy [4].

At certain combinations of the characteristics of the steam-generating channels, nearly sinusoidal oscillations of the steam parameters occur [1, 2]; this behavior is characteristic of self-oscillatory systems in the absence of a periodic external agent. It is also extremely undesirable, since it leads to the destruction of the steam generator.

In the present paper we take a different approach: we are concerned with the development of a system which converts a steady-state external agent into self-oscillation energy, which can, in turn, be converted into mechanical work (in a pump) or electrical energy [4].

Analysis of the Conditions for the Onset of Oscillations

Let us examine the conditions under which self-oscillations occur in a steam-generating system (Fig. 1) consisting of a hermetically sealed pipe of length $(2l_0 + l_1)$ and cross section S, filled with moist steam and containing a piston of mass M_1. Heat (Q) is supplied to the pipe through its ends, and heat is removed at a rate q through the lateral surface of the pipe; this latter rate depends on the properties of the steam. As mass leaves the system, an energy proportional to the mass velocity can be removed.

To determine the conditions for the onset of oscillations and to evaluate their frequency we write equations describing the relationship between the parameters of the systems under the following assumptions:

1) The moist steam is treated as an ideal gas with a heat of vaporization r = 0 (this condition allows us to write the quantitative relations between the parameters of the system in a more simple manner, without affecting the qualitative side of the problem).

2) The thermodynamic properties of the steam are homogeneous in the volumes under consideration.

Under these conditions the process can be described by

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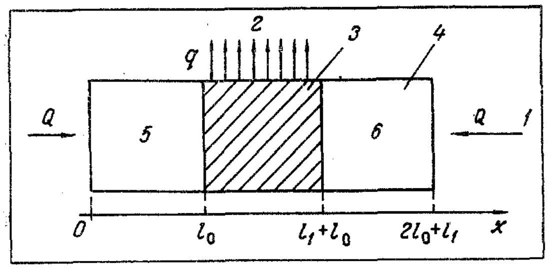

Fig. 1. Calculation model. 1) Heat source; 2) refrigerator; 3) inertial mass; 4) working medium; 5, 6) steam-filled volumes.

$$
\begin{aligned}
&P_1Sx = M_0RT_1,\\
&P_2S(2l_0 - x) = M_0RT_2,\\
&C_p\frac{dT_1}{dt} - \frac{RT_1}{P_1}\frac{dP_1}{dt} = \frac{Q-q_1}{M_0},\\
&C_p\frac{dT_2}{dt} - \frac{RT_2}{P_2}\frac{dP_2}{dt} = \frac{Q-q_2}{M_0},\\
&M_1\frac{d^2x}{dt^2} = (P_1 - P_2)S - A\frac{dx}{dt},
\end{aligned}
\tag{1}
$$

$$
q_1 = \begin{cases} 0, & 0 \leqslant x \leqslant l_0,\\ k\Pi(T_1 - T_3)(x - l_0), & l_0 \leqslant x \leqslant l_1 + l_0,\\ k\Pi(T_1 - T_3)l_1, & l_1 + l_0 \leqslant x \leqslant 2l_0,\end{cases}
$$

$$
q_2 = \begin{cases} k\Pi(T_2 - T_3)l_1, & 0 \leqslant x \leqslant l_0 - l_1,\\ k\Pi(T_2 - T_3)(l_0 - x), & l_0 - l_1 \leqslant x \leqslant l_0,\\ 0, & l_0 \leqslant x \leqslant 2l_0\end{cases}
$$

with the initial conditions

$$
P_1 = P_2 = P_3,\quad T_1 = T_2 = T_3,\quad x = l_0,\quad \frac{dx}{dt} = \frac{d^2x}{dt^2} = 0 \quad \text{at} \quad t = 0.
$$

In the absence of perturbations, $dx/dt = d^2x/dt^2 = 0$, we can write

$$
T_{10}(t) = T_{20}(t) = T_3 + \frac{Q}{M_0C_V}t,
$$

$$
P_{10}(t) = P_{20}(t) = P_3 + \frac{RQ}{Sl_0C_V}t.
$$

We now assume that at some time t_0 a perturbation $x = l_0 + \delta x$ is applied. Denoting the values of P and T at this time (taken as the zero of time) by P_0 and T_0, we find, after linearization of system (1) and some manipulations,

$$
a_3\frac{d^3\delta x}{dt^3} + a_2\frac{d^2\delta x}{dt^2} + a_1\frac{d\delta x}{dt} + a_0\delta x = 0,
\tag{2}
$$

where

$$
a_3 = \frac{VM_1}{(\varkappa - 1)M_0S};\quad a_2 = \frac{VA}{(\varkappa - 1)M_0S};\quad a_1 = \frac{2\varkappa P_0S}{M_0};
$$

$$
a_0 = \frac{k\Pi(T_0 - T_3)}{M_0} - \frac{2SQ(\varkappa - 1)}{VM_0};\quad V = Sl_0;\quad \varkappa = \frac{C_p}{C_V}.
$$

Oscillations can appear in the system if

$$
\begin{aligned}
&D = \zeta_1^3 + \zeta_2^2 > 0, \text{ where } \zeta_1 = \frac{1}{9}(3b_2^2 - b_1^2);\\
&\zeta_2 = \frac{1}{2}\left(\frac{2b_1}{27} - \frac{b_1b_2}{3} + b_3\right);\\
&b_1 = \frac{a_2}{a_3};\quad b_2 = \frac{a_1}{a_3};\quad b_3 = \frac{a_0}{a_3};
\end{aligned}
\tag{3}
$$

since here one root (K_3) of the characteristic equation in (2) is real, while two ($K_{1,2}$) are imaginary.

This system can thus be treated as a self-excited oscillatory system with a single degree of freedom. The appearance of oscillations in such a system is an internal property; no external agents depend on the time (in this case, the only external agents are the heating of the ends and the cooling of the lateral surface of the pipe).

We consider certain particular cases of Eq. (2).

1. We assume $M_1 = 0$, i.e., that the inertia of the piston is negligible. Then we have $a_3 = 0$ and $\mathrm{Re}K_{1,2} < 0$; in this case the oscillations are damped, and the equilibrium of the piston in the middle of the pipe is always stable.

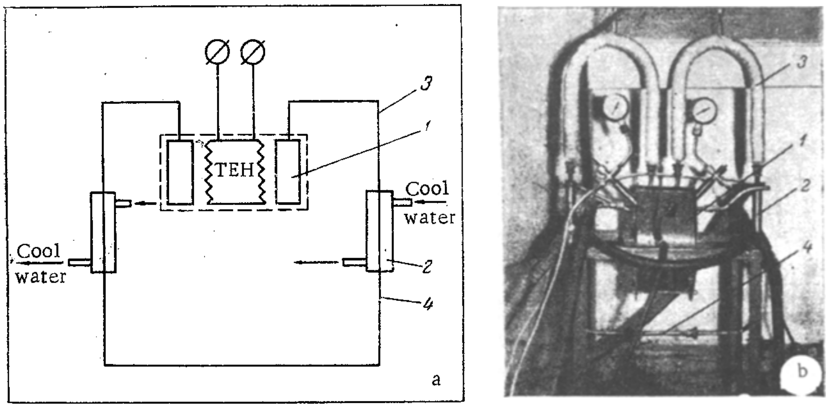

Fig. 2. Experimental apparatus. a) Block diagram; b) photograph. 1) Steam generator; 2) refrigerator; 3, 4) pipes.

2. We assume that there is no heat transfer, i.e., k = 0. Then $a_0 < 0$, and the roots K_i satisfy the conditions $K_1K_2K_3 = -a_0/a_3 > 0$ and $K_1 + K_2 + K_3 = -a_2/a_3 < 0$. Then if oscillations occur in the system (D > 0), the conditions $\operatorname{Re} K_1 = \operatorname{Re} K_2 < 0$ and $K_3 > 0$ must be satisfied. In other words, the oscillations are damped.

3. We assume that the motion of the piston is not retarded and that no power is drawn from the system, A = 0. Then

$$a_2 = b_1 = 0; \quad \zeta_1 = \frac{b_2}{3} > 0; \quad \zeta_2 = \frac{b_3}{2} \gtrless 0; \quad D > 0,$$

In other words, the motion is always oscillatory; it can be shown that in the case $\zeta_2 = 0$ the system is in a neutral state, in the case $\zeta_2 < 0$ the oscillations in the system are damped, while in the case $\zeta_2 > 0$ the oscillations in the system grow.

Determination of the Frequency and Amplitude of the Oscillations

We are not claiming that these evalutions of the frequency and amplitude of the oscillations constitute an exhaustive description of the phenomena. This analysis is based on a simplified physical model of the processes occurring in a system in which the thermodynamic medium itself is used as the mass M_1. Furthermore, we are treating the case A = 0. A steam generator of volume V is connected by a pipe of length l_2 and cross section $S (V \gg S l_2)$ with a refrigerator, and the liquid level in the system is at a distance l_3 from the steam generator. We assume that the oscillations in the system are sinusoidal, $x = x_0 \cdot \sin(2\pi t/\tau)$; this assumption is verified by experiment (Fig. 3a).

In the steady state, the following conditions should hold:

1) The maximum pressure drop in the system is governed by the energy stored in the steam generator over a half-period $\tau/2$:

$$|\Delta P|_{\max} S \equiv |P_1 - P_2|_{\max} S \simeq \frac{Q\tau R T_0 S}{2V\,[r_0 + C_p (T_0 - T_3)]} \simeq M_1 \left(\frac{d^2x}{dt^2}\right)_{\max} \simeq M_1 x_0 \left(\frac{2\pi}{\tau}\right)^2, \tag{4}$$

where r_0 is the heat of vaporization of the liquid at temperature T_0. Here we are assuming that the liquid is at temperature T_3 before it enters the steam generator and that the steam has the properties of an ideal gas.

2) The power Q supplied to the system is dissipated in the refrigerator:

$$Q = k\,(T_0 - T_3)\,\Pi\,(x_0 - l_2 + l_3); \tag{5}$$

3) The oscillation amplitude x_0 can be evaluated from the condition for evaporation of the liquid ΔM which enters the steam generator:

$$\frac{Q\tau}{2} = \Delta M\,[r_0 + C_p (T_0 - T_3)] \simeq [r_0 + C_p (T_0 - T_3)]\, S\,(x_0 - l_3)\,\rho. \tag{6}$$

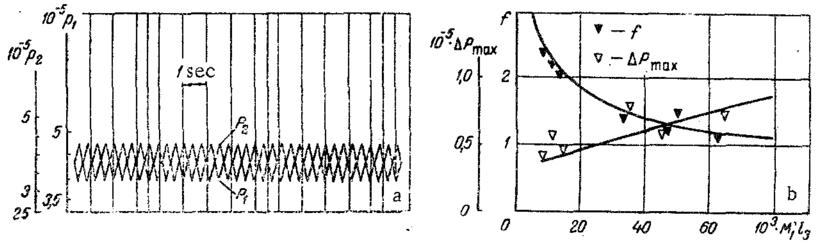

Fig. 3. Experimental results for Q = 800 W. a) Oscillogram of the pressure in the steam generators; b) experimental and calculated dependences of the frequency f (Hz) and the amplitude of the pressure oscillations, $|\Delta P|_{max}$ (N/m^2), in the steam generators as functions of the parameter $M_1 l_3$ (kg · m).

Equations (4)-(6) relate the quantities τ, $|\Delta P|_{max}$, T_0, and x_0 with the governing parameters of the system (R, C_p, S, Q, T_3, M_1, r_0, V, Π, l_3, l_2).

Using the relation

$$r_0 + C_p (T_0 - T_3) \simeq \text{const} \equiv C_1, \tag{7}$$

which holds for most liquids over a broad range, we can find from (4)-(7) the following dependence of the oscillation frequency $f = 1/\tau$ on the governing parameters of the system:

$$f^3 = \frac{QRS\left[\dfrac{Q}{k\Pi\left(\dfrac{Q}{2C_1 S\rho f} + 2l_3 - l_2\right)} + T_3\right]}{(2\pi)^2\, 2VC_1 M_1\left(\dfrac{Q}{2C_1 S\rho f} + l_3\right)}. \tag{8}$$

In the case $Q/2C_1 S\rho f \ll l_3$ Eq. (8) becomes

$$f \simeq \left[\frac{RSQ\left(T_3 + \dfrac{Q}{kF}\right)}{2(2\pi)^2 C_1 V M_1 l_3}\right]^{\frac{1}{3}}, \tag{9}$$

where F is the heat-transfer surface area of the refrigerator. The amplitude of the pressure oscillations in the system in this case is described by

$$|\Delta P_{max}| = |P_1 - P_2|_{max} = \frac{\left(T_3 + \dfrac{Q}{kF}\right) QR}{2C_1 V f}. \tag{10}$$

Experimental Apparatus and Results

To study self-oscillations in this system we assembled an apparatus (Fig. 2) consisting of two steam generators 1, two refrigerators 2, and connecting pipes 3 and 4. All the pipes have an inside diameter of $d = 8 \cdot 10^{-3}$ m. The steam generator consists of a flat tank and an electric heating element, pressed tightly together. Points on the tanks are provided for mounting thermocouples and pressure gauges. The steam generators are enclosed in a common heat-insulation jacket; the pipes connecting the steam generators with the refrigerators are also insulated. The refrigerators are of the pipe-in-pipe type, with the surface of the inner tube finned; the cooling of the refrigerators is sequential.

In the experiments we measured the following: 1) the static pressure $\bar{P}$ and the pressure oscillations $|\Delta P|_{max}$ in the steam generators ($\bar{P} \leq 10^6$ N/m^2; $|\Delta P|_{max} \leq 10^5$ N/m^2); 2) the steam temperature in the steam generators, $T_0 \leq 500$°K; 3) the flow rate of cooling water ($\simeq 4 \cdot 10^{-2}$ kg/sec); 4) the heating of the cooling water (≤ 10°K).

The pressure measurements are carried out with DD-10 pressure gauges, which are capable of detecting pressure changes at frequencies up to 1000 Hz. The results are recorded on a loop oscillograph. The error of these pressure measurements is less than ±3% of the measured values. The frequency of the pressure oscillations is determined from time markers on the oscillograph. The error in the markers is less than 1% and the error of the frequency determination is less than 5%.

The vapor temperature is measured with Chromel— Copel thermocouples located in the steam phase of the tanks. These results are recorded by an ÉPP-09-M3 recording potentiometer. The error in the determination of the temperatures in the tanks is less than ±3.5°C.

The warming of the water is determined by a differential thermocouple with 20 junctions, 10 of which are at the entrance to the refrigerator and 10 at the exit from it. The corresponding error is less than ±0.3°C.

The flow rate of the cooling water is monitored with a turbine flowmeter at the exit from the refrigerator. The flow rate is determined from the indications of the frequency meter; the corresponding error is less than 6%.

In the experiments we achieved regimes with stable oscillations. The oscillograms of the pressure changes in the system (Figs. 3a) show that these oscillations are approximately sinusoidal and that the pressure changes in the steam generators are out of phase. The maximum amplitude of the pressure oscillations observed in these experiments was $1 \cdot 10^5$ N/m^2, and the frequency ranged from 1 to 4 Hz.

Figure 3a shows a typical pressure oscillogram; Fig. 3b shows the calculated and experimental dependences of the frequency f and the amplitude $|\Delta P|_{max}$ of the pressure oscillations in the system on the quantity $M_1 l_3$, varied in the experiments. This quantity was varied by filling the connecting pipe 4 with various amounts of water [$M_1 = (20\text{–}280) \cdot 10^{-3}$ kg] and by changing the length of pipe 3 [$l_2 = (0.05\text{–}0.25)$ m, $l_2 \simeq l_3$].

The calculated curves were found from Eqs. (9) and (10) on the basis of the following initial data: $Q = 8 \cdot 10^2$ W; $R = 4.7 \cdot 10^2$ J/(kg·deg), $C_1 = 2.6 \cdot 10^6$ J/kg, $S = 0.5 \cdot 10^{-4}$ m^2, $\Pi = 10^{-1}$ m, $k = 2 \cdot 10^3$ W/(m^2·deg) (the steam — water heat-transfer coefficient in the refrigerator is governed by the wall — water heat-transfer coefficient for a flow rate of $4 \cdot 10^{-2}$ kg/sec of the cooling water in an annular gap $d \simeq 10^{-2}$ m in diameter), $\delta = 2 \cdot 10^{-3}$ m, $T_3 = 3 \cdot 10^2$°K, $F = 1.5 \cdot 10^{-2}$ m^2, and $V = 2.5 \cdot 10^{-4}$ m^3.

NOTATION

Q	is the heat flux;
q	is the specific (linear) heat flux;
T, P	are the gas temperature and pressure;
R	is the universal gas constant;
C_p, C_V	are the specific heats at constant pressure and at constant volume;
Π, F	are the cooled perimeter and surface area of the refrigerator;
T_3	is the temperature of the refrigerator walls;
$P_3 = P(T = T_3)$, T_0	are the steady-state steam temperature in the steam generator;
k	is the heat-transfer coefficient;
ρ	is the liquid density;
r	is the heat of vaporization of the liquid;
M_0	is the gas mass in volume 1 or 2;
S, M_1	are the area and mass of piston;
A	is the power drawn from the apparatus with $dx/dt = 1$;
x	is the coordinate;
t	is the time;
τ	is the oscillation period.

Subscripts

"1" and "2" refer to volumes 1 and 2, respectively.

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